# Workshop
# THEoretical Studies on IncentiveS

## “The Optimal Degree of Discretion in Monetary Policy”

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Econometrica, 2005

How much discretion should the monetary authority have in setting its policy?

- the central bank has some private knowledge about the economy.
- tension between discretion and the incentive problem of the central bank.
- the incentive problem is modeled as a time inconsistency problem in which the central bank is tempted to claim that the current state of the economy justifies a monetary stimulus to output (through unexpected inflation).

What do we observe in reality?

- Norges Bank: about 2.5% ($\pm$1%)
- Sveriges Riksbank: 2% ($\pm$1%)
- RBNZ: between 1 and 3 percent
- Bank of England: inflation target of 2% ($\pm$1%)
- The US Federal Reserve's policy setting committee and its members, regularly publicly state a desired target range for inflation (usually around $1.5 - 2\%$), but do not have an explicit inflation target.

Preview of the results:

- As long as a monotone hazard condition is satisfied, the optimal mechanism is static.
- The optimal policy has one of two forms:
  - bounded discretion
  - no discretion
- The optimal policy can be implemented as a repeated static equilibrium of a game in which
  - the monetary authority chooses its policy subject to an inflation cap
  - the agent's expectations of future inflation do not vary with the monetary authority's policy choice

The Model:

- $t = 0, 1, \ldots$
- Monetary authority and a continuum of individual agents
- At the beginning of each period, agents choose individual action $z_t$ from some compact set.
  $z_t$: growth rate of an individual's nominal wage.
  $x_t$ : growth rate of the average nominal wage
- The monetary authority observes $\theta_t$.
  $\theta_t$ is i.i.d, mean 0 r.v. with support $[\underline{\theta}, \bar{\theta}]$ and strictly positive density $p(\theta)$ and a c.d.f $P(\theta)$
- Given $\theta$, the monetary authority chooses money growth $\mu_t$ in some large compact set $[\underline{\mu}, \bar{\mu}]$

- The monetary authority maximizes a social welfare function $R(x_t, \mu_t, \theta_t)$ which is strictly concave in $\mu$ and twice continuously differentiable.
- In terms of current payoffs, the monetary authority prefers to choose higher inflation when higher values of this state are realized and lower inflation when lower values are realized: $R_{\mu\theta}(x_t, \mu_t, \theta_t) > 0$.
- A *mechanism* specifies what monetary policy is chosen each period as a function of the history of the monetary authority's reports of its private information.
- If $x = \int \mu(\theta)p(\theta)d\theta$, then $\int R_x(x, \mu(\theta), \theta)p(\theta)d\theta < 0$

Definitions:

- A *policy* for the monetary authority in any given period, $\mu()$, specifies the money growth rate $\mu(\theta)$ for each $\theta$,
- For any $x$, the static best response $\mu^*(\theta, x)$ solves $R_\mu(x, \mu(\theta), \theta) = 0$

Ramsey Benchmark:

1. Ramsey policy, $\mu^R()$, yields the highest payoff that can be achieved in an economy with full information.

$$\max_{x,\mu()} \int R(x, \mu(\theta), \theta) p(\theta) d\theta$$
$$\text{subject to } x = \int \mu(\theta) p(\theta) d\theta$$

2. Expected Ramsey policy, $\mu^{ER}$, is the optimal policy when the policy is restricted to not depend on private information.

$$\max_{x,\mu} \int R(x, \mu, \theta) p(\theta) d\theta$$
$$\text{subject to } x = \mu$$

From the Revelation Principle, the problem we need to solve is

$$\max_{\mu(.)}(1-\beta)\sum_{t=0}^{\infty}\int \beta^t R(x_t,\mu_t(\theta_t),\theta_t)p(\theta_t)d\theta_t$$

Subject to

$$z_t = \int \mu_t(\theta)p(\theta)d\theta$$

$$x_t = z_t$$

The monetary policy is incentive-compatible (IC)

Recursive method (Abreu, Pearce and Stachetti (1990))

- The repeated nature of the model implies that the set of IC payoffs that can be obtained from any period $t$ can be obtained from period 0.
- So the payoff from any IC outcome can be broken down into payoffs from current actions and continuation payoffs (also in the set of IC payoffs)
- For each possible report $\hat{\theta}$, there is a corresponding continuation payoff $w(\hat{\theta})$ that represents the discounted utility for the monetary authority from the next period on.

Definition: $x$, $\mu()$ and $w()$ are **enforcable** by the candidate set of IC levels of social welfare ($W$) if

$$w(\hat{\theta}) \in W \text{ for all } \hat{\theta} \in [\underline{\theta}, \bar{\theta}]$$
$$x = \int \mu(\theta)p(\theta)d\theta$$
$$(1-\beta)R(x, \mu(\theta), \theta) + \beta w(\theta) \geq (1-\beta)R(x, \mu(\hat{\theta}), \theta) + \beta w(\hat{\theta})$$

Under the single-crossing assumption we can replace the global IC constraint by:

- $\mu(.)$ nondecreasing
- $$(1-\beta)R_\mu(x, \mu(\theta), \theta)\frac{d\mu(\theta)}{d\theta} + \beta\frac{dw\theta}{d\theta} = 0$$

Define

$$U(\theta) = R(x, \mu(\theta), \theta) + w(\theta)$$

$$U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} \frac{1 - P(\theta)}{p(\theta)} R_\theta(x, \mu(\theta), \theta) p(\theta) d\theta$$

and

$$U(\theta) = U(\bar{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} \frac{P(\theta)}{p(\theta)} R_\theta(x, \mu(\theta), \theta) p(\theta) d\theta$$

Rewriting the maximization problem:

$$\max U(\underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} \frac{1 - P(\theta)}{p(\theta)} R_\theta(x, \mu(\theta), \theta) p(\theta) d\theta$$

subject to
$(i)$ $x = \int \mu(\theta) p(\theta) d\theta$
$(ii)$ $\mu(\theta)$ is non decreasing
$(iii)$ $w(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} R_\theta(x, \mu(z), z) dz - R(x, \mu(\theta), \theta)$ where
$w(\theta) \leq \bar{w}$

Or alternatively

$$\max U(\bar{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}} \frac{P(\theta)}{p(\theta)} R_\theta(x, \mu(\theta), \theta) p(\theta) d\theta$$

subject to $(i)$, $(ii)$ and
$(iii)$ $w(\theta) = U(\bar{\theta}) - \int_{\theta}^{\bar{\theta}} R_\theta(x, \mu(z), z) dz - R(x, \mu(\theta), \theta)$ where
$w(\theta) \leq \bar{w}$

**Proposition (1)**

*Under a single-crossing condition (A1: $R_{\mu\theta}(x, \mu, \theta) > 0$) and a "monotone hazard condition", the optimal mechanism is static ($w(\theta) = \bar{w}$).*

Monotone hazard condition:

(A2a) $\frac{1-P(\theta)}{p(\theta)} R_{\theta\mu}(x, \mu(\theta), \theta)$ is strictly decreasing in $\theta$

(A2b) $\frac{P(\theta)}{p(\theta)} R_{\theta\mu}(x, \mu(\theta), \theta)$ is strictly increasing in $\theta$

**Sketch of the proof of Proposition 1:**
Consider $\mu(.)$ which is increasing on $(\theta_1, \theta_2)$ and

$$\tilde{\mu}(\theta) = \begin{cases} \tilde{\mu} & \text{if } \theta \in (\theta_1, \theta_2) \\ \mu(\theta) & \text{otherwise} \end{cases}$$

Where $\tilde{\mu}$ is the conditional mean of $\mu(.)$ on $(\theta_1, \theta_2)$.

$$\mu(\theta; a) = a\tilde{\mu}(\theta) + (1 - a)\mu(\theta)$$

Where $a \in [0, 1]$.

The expected inflation remains unchanged with this variation.

Need also to ensure feasibility ($w(\theta; a) \leq \bar{w}$ for all $\theta$) and IC

Can ensure feasibility with the following adjustments of the continuation values

Up variation:

$w(\theta; a) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} R_\theta(x, \mu(z; a), z)dz - R(x, \mu(\theta; a), \theta)$

Down variation:

$w(\theta; a) = U(\bar{\theta}) - \int_{\theta}^{\bar{\theta}} R_\theta(x, \mu(z; a), z)dz - R(x, \mu(\theta; a), \theta)$

- Lemma 1: Let $(x, \mu(.), w(.))$ be an allocation in which $\mu(.)$ is increasing on some interval $(\theta_1, \theta_2)$. Then the up variation and the down variation both improve welfare by increasing the objective function.
- Lemma 2: In the optimal mechanism, the continuation value function $w(.)$ is a step function.
- Lemma 3: $\mu(.)$ and $w(.)$ are continous.

Characterizing the optimal (static) policy:

$$\max_{\mu()} \int R(x, \mu(\theta), \theta) p(\theta) d\theta$$

subject to

$$x = \int \mu(\theta) p(\theta) d\theta$$

$$R(x, \mu(\theta), \theta) \geq R(x, \mu(\hat{\theta}), \theta)$$

Useful definitions:

- Bounded discretion

$$\mu(\theta) = \left\{ \begin{array}{ll} \mu^*(\theta; x) & \text{if } \theta \in [\underline{\theta}, \theta^*) \\ \mu^* = \mu^*(\theta^*; x) & \text{if } \theta \in [\theta^*, \bar{\theta}] \end{array} \right\}$$

- No discretion

$$\mu(\theta) = \mu \text{ for some constant } \mu$$

**Proposition (2)**

*Under assumptions (A1) and (A2), the optimal policy $\mu()$ has either bounded discretion or no discretion.*

- The optimal policy can be implemented by society by setting an upper limit on the inflation rate which the monetary authority is allowed to choose.
- The optimal degree of discretion is decreasing in the severity of the time inconsistency problem.

**Intuition/Proof of Proposition 2:**

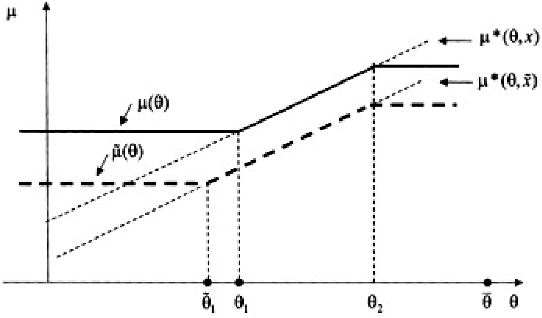


FIGURE 4.—An alternative welfare-improving policy variation.

**Proposition (3)**

*Under assumptions (A1) and (A2),*

- *If the static best response satisfies* $\mu^*(\underline{\theta}, x) \geq x$ *for all* $x \in [\underline{\mu}, \bar{\mu}]$*, then the optimal policy has no discretion.*
- *If the static best response satisfies* $\mu^*(\underline{\theta}, \mu^{ER}) < \mu^{ER}$*, then the optimal policy has bounded discretion.*

Conclusion:

- The optimal mechanism is static.
- The monetary authority also maximizes the welfare of society, only conflict related to the time inconsistency problem.
- The optimal policy has either bounded discretion or no discretion.