

Workshop
THEoretical Studies on IncentiveS

November 4, 2008

“Relying on the information of interested parties”
by Paul Milgrom and John Roberts (Rand 1986)

Real world problem:

- Decisionmakers need to rely on information provided by individuals who are affected by their decisions.
- Although interested parties may try to manipulate the decisionmaker's choice by concealing or distorting information, their efforts do not always succeed.

Question to be answered:

Does competition among interested parties attempting to influence a decisionmaker by providing verifiable information elicit all relevant information?

Or more generally, when can the decisionmaker elicit all relevant information?

Persuasion Game

The DM and the interested party/parties interact only once.

⇒ Issues of reputation do not arise

3 cases considered

- 1 One seller and one sophisticated buyer.
- 2 Competition among interested parties.
- 3 Competition among interested parties with limited communication.

One seller and one sophisticated buyer

- Nature selects the seller's information x from a finite set X .
- The buyer's beliefs about $x \in X$ are described by $P(x) > 0$.
- The seller observes x and makes an assertion $A \in X$ to the buyer.
Restriction: $x \in A$.
- The buyer observes A and selects a quantity $q \in R^+$.
- The payoffs $u(x, q)$ to the buyer and $v(x, q)$ to the seller are realized.

Equilibrium concept: A sequential equilibrium (Kreps and Wilson (82)) is a triple (r, b, p) , where r and b are the reporting and buying strategies, respectively, and p specifies what the buyer believes when the seller makes a report.

The triple is a sequential equilibrium in pure strategies if

- (i) Seller maximization: r is the seller's best response to b .
- (ii) Buyer maximization: For all A , $b(A)$ is the best purchase for the buyer given his beliefs.
- (iii) Rational buyer expectations.
- (iv) Consistent beliefs.

Definitions:

- A *posture* is a pair of (b, p) satisfying (ii) and (iv).
- A *naively credulous posture* is such that $p(x|A) = P(x)/P(A)$.
- A *skeptical posture* (\bar{b}, \bar{p}) is such that for all A , $(\bar{b}(A), \bar{p})$ solves

$$\min_{Q,p} Q$$

subject to

$$Q \text{ maximizes } \sum_x u(x, q)p(x|A)$$

$p(\cdot|\cdot)$ a conditional probability

$$p(x|A) = 0 \text{ for all } x \notin A$$

Proposition 1

If the buyer's posture (b, p) is a skeptical posture and the full-information decision is always reached, the (r, b, p) is an equilibrium.

Proposition 2

If the buyer's utility function is strictly concave and continuously differentiable in q , then at every equilibrium the buyer adopts a skeptical posture (b, p) and always purchases the full-information quantity.

Competition among interested parties

Structure of the persuasion game:

- 1 Nature chooses x according to the distribution P .
- 2 Each of the N interested parties observes x .
- 3 Simultaneously, each interested party suggested a set D_i of possible decisions d , chosen from a finite set Δ , and asserts a true proposition.

Proposition 4

Suppose that for every x and every decision d in $\Delta - \{f(x)\}$ there is some interested party who prefers the full-information decision $f(x)$ to d . Then at every pure-strategy Nash equilibrium, the full-information decision is taken.

Sellers and buyer example

Each seller has a product to offer whose cost of production is $c_i(x)$. If a sale is concluded at price p , the seller's payoff is $p - c_i(x)$ and the buyer's utility is $u(x, i) - p$. Moving simultaneously, sellers name prices p_i for their products and make reports $r_i(x)$. The buyer selects one of the products to purchase and payoffs are realized.

Corollary 2

At every pure-strategy Nash equilibrium of the persuasion with strongly opposed interests, the equilibrium decision is the full-information decision.

Corollary 3

At every pure-strategy Nash equilibrium of the persuasion and pricing game, the equilibrium choice and price are the same as in the corresponding full-information price-setting game.

What does this *really* mean?

When the interested parties are fully informed and able to report all they know and when the full information decision is Pareto optimal for them, competition in suggesting alternatives and providing information can obviate the need for the DM to be well informed and sophisticated.

Competition and sophistication

Relax the assumption that all interested parties can report all they know, but reintroduce sophistication.

- Nature chooses a triplet $\omega = (Z, x, y)$ according to some probability distribution P .
- Each party observes ω and simultaneously reports D_i and A_i to the DM.
- The DM then chooses a decision d from the suggested alternatives and the payoffs are $u(Z, x_d, d)$ and $v_i(\omega, d)$.

Competition and sophistication (cont'd)

Proposition 5

Suppose that for no ω and no $d \in \Delta$ is it true that d is preferred or indifferent to the full-information decision at ω by all parties. Then there exists a sequential equilibrium at which the DM adopts the skeptical strategy described above. At every such equilibrium, the decision reached is the full-information decision.