Workshop
THEoretical Studies on IncentiveS

October 19, 2009

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"The Swing Voter's Curse"

by Feddersen and Pesendorfer (AER 1996)

The Real World

1994 election, Illinois:

Registered voters:	6,119,011
Voters in gubernatorial race:	3,106,566
Voters constituional amendment:	2,144,200

Stylized fact: better educated and informed individuals are more likely to participate in an election that less educated and informed individuals (Wolfinger & Rosenstone (1980)).

Objective

Show that:

- Informational asymmetries may also influence both participation and vote choice independent of costs to vote and pivot probabilities.
- Less informed voters have an incentive to delegate their vote via abstention to more informed voters.

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- Similarity between the winner's curse in auction theory and the swing voter's curse.
- Swing voter = agent whose vote determines the outcome of an election.
- Both in auctions and elections an agent's action only matters in particular circumstances.
- When other agents have private info that may be useful to an agent, the agent must condition his action, not only on his info, but also on what must be true about the world if his action matters.

Example: Swing voter's curse

- Two candidates: Status quo (0) and an alternative (1).
- Voters are uncertain about the cost of implementing alternative: high in state 0 or low in state 1.
- If cost is high (proba $= 0.9$) , everyone prefer the status quo.
- If cost is low (proba $= 0.1$), everyone prefer the alternative.
- At least one voter is informed and knows the cost with certainty.
- Voters do not know the exact number of informed voters.

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  - Informed voters vote for 0 in state 0 and 1 in state 1.
  - Uninformed voters vote for 0 in both states.
- This is not rational for the uninformed voters!!

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Example: why it is not rational

- An uninformed voter is only pivotal if some voters have voted for the alternative.
  $\Rightarrow$ Informed voters voted for the alternative (i.e. state $= 1$)
  $\Rightarrow$ Uninformed voters should vote for the alternative.
- For the same reason, it cannot be rational for all uninformed voters to vote for the alternative.

So, it is not optimal for the uninformed voters to vote only on the basis of their prior info.

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Here, easy solution: uninformed voters abstain!
Uniformed voters suffer the swing voter's curse, i.e. they are better of abstaining.

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- This paper formalizes and extends this result to include voters with different preferences:
  - 0-partisans always prefer candidate 0 (status quo)
  - 1-partisans always prefer candidate 1 (the alternative)
  - Independent voters prefer 0 in state 0 and 1 in state 1
- All voters know the expected percentage of each type within the population but not the exact number.
- For all voters, proba of knowing the state is strictly positive.

Main Result

- If no agent uses a strictly dominated strategy then uninformed voters who are almost indifferent between voting for either of the two candidates suffer the swing voter's curse and are strictly better off by abstaining.
- For a wide range of parameters a significant fraction of voters abstain in large elections.
- When voters behave strategically, large elections under private information, almost always choose the same winner as would be chosen by a fully informed electorate.

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The model

- States: $\mathcal{Z} = \{0, 1\}$.
- Candidates: $\mathcal{X} = \{0, 1\}$.
- Types of agents: $\mathcal{T} = \{0, 1, i\}$.
    - Type 0 always strictly prefer candidate 0.
    - Type 1 always strictly prefer candidate 1.
    - Type i prefer 0 in state 0 and 1 in state 1.

$$U_i(x, z) = \begin{cases} -1 & \text{if } x \neq z \\ 0 & \text{if } x = z \end{cases}$$

- At the beginning of the games, nature chooses a state $z \in \mathcal{Z}$. $\Pr(\mathcal{Z} = 0)=\alpha$ ($\leq .5$ w.l.o.g).
- Nature chooses a set of agents by N+1 independent draws.

Model - More on the agents

- There is uncertainty about the total number of agents and the number of agents of each type.
- Proba that nature selects an agent: $1 - p_\Phi$.
- If an agent is selected, then she is of type
  - i with proba $\frac{p_i}{1-p_\Phi}$
  - 0 with proba $\frac{p_0}{1-p_\Phi}$
  - 1 with proba $\frac{p_1}{1-p_\Phi}$

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  - 0 with proba $\frac{p_0}{1-p_\Phi}$
  - 1 with proba $\frac{p_1}{1-p_\Phi}$
- After the state and set of agents have been chosen, every agent learns her type and receives a message $m \in \mathcal{M} = \{0, \alpha, 1\}$, which is the proba of state 0.
- Type and message are private info.
- All informed agents receive the same message.
- Proba of an agent being informed is q.

Strategies and Equilibrium

- Every agent chooses an action $s \in \{\Phi, 0, 1\}$.
- The candidate that receives a majority of the votes cast is elected.
- Focus on symmetric Nash Equilibria (NE).
- Number of agents uncertain and ranges from 0 to N+1
  $\Rightarrow$ Proba of an agent being pivotal is strictly positive.
- All agents except the uninformed independent agents (UIAs) have a strictly dominant strategy (and will play these in equilibrium).
- Therefore, focus on the behavior of UIAs.

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UIA Analysis

- Denote a mixed strategy profile $\tau = (\tau_0, \tau_1, \tau_i) \in [0,1]^3$.
- For a given τ, define $\sigma_{z,x}(\tau)$ to be the proba that a random draw by nature results in a vote for candidate x if the state is z:

$$\sigma_{z,x}(\tau) = \begin{cases} p_x + p_i(1-q)\tau_x & \text{if } x \neq z \\ p_x + p_i(1-q)\tau_x + p_i q & \text{if } x = z \end{cases}$$

- Similarly $\sigma_{z,\Phi}(\tau)$ is the proba that a random draw does not result in a vote for either candidate:

$$\sigma_{0,\Phi}(\tau) = \sigma_{1,\Phi}(\tau) = \sigma_{\Phi}(\tau) = p_i(1-q)\tau_\Phi + p_\Phi.$$

UIA Analysis (cont'd)

When is an UIA pivotal	Probability
Equal number of other agents vote for each candidate	$\Pi_t(z, \tau)$
1 receives one less vote than 0	$\Pi_1(z, \tau)$
0 receives one less vote than 1	$\Pi_0(z, \tau)$

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Define $Eu(x, \tau)$ - the expected payoff to an UIA when taking action x and the strategy of other players being τ.

$$\begin{aligned} Eu(1,\tau) - Eu(\Phi,\tau) = \frac{1}{2}[(1-\alpha)[\Pi_t(1,\tau) + \Pi_1(1,\tau)] \\ -\alpha[\Pi_t(0,\tau) + \Pi_1(0,\tau)]] \end{aligned} \tag{1}$$

$$\begin{aligned} Eu(0,\tau) - Eu(\Phi,\tau) = \frac{1}{2}[\alpha[\Pi_t(0,\tau) + \Pi_1(0,\tau)] \\ -(1-\alpha)[\Pi_t(1,\tau) + \Pi_1(1,\tau)]] \end{aligned} \tag{2}$$

$$\begin{aligned} Eu(1,\tau) - Eu(0,\tau) = (1-\alpha) \times [\Pi_t(1,\tau) + 1/2(\Pi_1(1,\tau) \\ +\Pi_0(1,\tau))] - \alpha[\Pi_t(0,\tau) + 1/2(\Pi_1(0,\tau) + \Pi_0(1,\tau))] \end{aligned} \tag{3}$$

Proposition 1

Let $p_\Phi > 0$, $q > 0$, $N \geq 2$ and N even. For any symmetric strategy profile τ in which no agent plays a strictly dominated strategy, $Eu(1,\tau) = Eu(0,\tau)$ implies $Eu(1,\tau) < Eu(\Phi,\tau)$.

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"Proof": $Eu(1,\tau) = Eu(0,\tau)$ + Equations (3) and (1):

$$Eu(1,\tau) - Eu(\Phi,\tau) = 1/4[(1-\alpha)[\Pi_1(1,\tau) - \Pi_0(1,\tau)] + \alpha[\Pi_0(0,\tau) - \Pi_1(0,\tau)]] \quad (4)$$

+ Definitions of probabilities previously presented:

$$Eu(1,\tau) - Eu(\Phi,\tau) < 0.$$

Voting & Participation in Large Elections

In this section, we will define a sequence of games and show that:

- The swing voter's curse (Prop 1) can lead to large scale abstention by UIAs. This abstention is unrelated to the fact that the proba of being pivotal is very small in large elections.
- Equilibrium voting behavior virtually guarantees that the winning candidate is the same as the candidate that would win if voters had perfect info.

Lemma 1

Suppose $p_i q > 0$ and $0 < \alpha < 1$. Consider a sequence of voting games and strategy profiles $\{\tau^N\}_{N=0}^{\infty}$. Then:

A. If there exists an $\varepsilon > 0$ such that $\sigma_{x,y}(\tau^N) - \sigma_{y,x}(\tau^N) > \varepsilon$ for any $N \geq 0$ and $x \neq y$ then there exists an $\bar{N}$ such that for any $N > \bar{N}$ $Eu(x, \tau^N) > Eu(\Phi, \tau^N) > Eu(y, \tau^N)$.

B. If for all $N \geq 0$ there are two actions s, s' with $s \neq s'$ such that $Eu(s, \tau^N) = Eu(s', \tau^N)$ then for any $\varepsilon > 0$ there is an $\bar{N}$ such that for $N > \bar{N}$ $|\sigma_{0,1}(\tau^N) - \sigma_{1,0}(\tau^N)| < \varepsilon$.

Intuition Lemma 1A:

- If $\sigma_{1,0}(\tau^N) - \sigma_{0,1}(\tau^N) > \varepsilon$, then the conditional proba that the world is in state 1 given the agent is pivotal goes to 1 as the size of the electorate increases.
- This follows from the fact that an agent is only pivotal if enough agents make a mistake to compensate for the votes of the informed independent agents (IIA).
- If the proba of a mistake is higher in state 1 than in state 0 then an UIA is more likely to be pivotal in state 1 than state 0 and he strictly prefers to vote for candidate 1 rather than abstain and would rather abstain than vote for 0.

- UIAs do not know the state with certainty and therefore are unsure of the candidate that they prefer.
- On the other hand UIAs would always prefer that the informed independent agents (IIAs) decide the election.
- The effect of equilibrium behavior of the UIAs is to maximize the proba that the IIAs determine the winner.
- UIAs vote to compensate for the partisans and having achieved that compensation they abstain.

Case when UIAs can't compensate for partisan advantage

Proposition 2

Suppose $q > 0$, $p_i(1-q) < |p_0 - p_1|$ and $p_\Phi > 0$. Let $\{\tau^N\}_{N=0}^{\infty}$ be a sequence of equilibria.

(i) If $p_i(1-q) < p_0 - p_1$ then $\lim_{N\to\infty} \tau_1^N = 1$, that is, all UIAs vote for candidate 1.

(ii) If $p_i(1-q) < p_1 - p_0$ then $\lim_{N\to\infty} \tau_0^N = 1$, that is, all UIAs vote for candidate 0.

Proof: Immediate consequence of Lemma 1A.

Case where the UIAs can fully offset the partisan bias

Proposition 3

Suppose $q > 0$, $p_i(1-q) \geq |p_0 - p_1|$ and $p_\Phi > 0$. Let $\{\tau^N\}_{N=0}^{\infty}$ be a sequence of equilibria.

(i) If $p_i(1-q) \geq p_0 - p_1 > 0$ then UIAs mix between voting for candidate 1 and abstaining; $\lim \tau_1^N = (p_0 - p_1)/p_i(1-q)$ and $\lim \tau_\Phi^N = 1 - [(p_0 - p_1)/p_i(1-q)]$.

(ii) If $p_i(1-q) \geq p_1 - p_0 > 0$ then UIAs mix between voting for candidate 0 and abstaining; $\lim \tau_0^N = (p_1 - p_0)/p_i(1-q)$ and $\lim \tau_\Phi^N = 1 - [(p_1 - p_0)/p_i(1-q)]$.

(iii) If $p_0 - p_1 = 0$ then UIAs abstain; $\lim \tau_\Phi^N = 1$.

Proof: Immediate consequence of Proposition 1 and Lemma 1.

Information Aggregation

Proposition 4

Suppose $p_\Phi > 0$ and $q > 0$ and $p_i \neq |p_0 - p_1|$. Then for every ε there exists an $\bar{N}$ such that for $N > \bar{N}$ the probability that in equilibrium the election fully aggregates information is greater than $1 - \varepsilon$.

Proof: Propositions 2 and 3 + Law of Large Numbers.

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