Costly Distortion of Information in Agency Problem

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In agency relationship, agents often spend resources distorting information transmitted to principals: regulated firms, taxpayers, insured agents...

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  - the principal may have a strict preference for mechanisms that give rise to falsification,
  - resources are spent in falsification activities.
- Research question: How costly information distortion can emerge as equilibrium behavior?

- Approach: Use of the extended standard principal-agent model introducing more general information structure, i.e., the agent can distort at a cost, the signal received by the principal about the state of nature.

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- Results: When information is partially private, inducing falsification can help the principal reduce information rents.

Plan

- The Model

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- Conclusion

- The principal's utility: $U^P(q,y) = V(q) - y$, with $V'(q) > 0$ and $V''(q) < 0$.

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- The agent privately observes θ before signing the contract.

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- *Assumption 1*: $c(e) \geq 0$; $c(\theta) = 0$; $c(e)$ twice continuously differentiable; $c"(e) > 0$.
- The agent's utility: $U(q, s, y, \theta) = y - \theta q - c(s - \theta)$,

By the revelation principle, the optimal allocation $\{q(\theta), s(\theta), y(\theta)\}$ solves:

$$\max_{q(\theta),s(\theta),y(\theta)} E[U^P(q(\theta), y(\theta))],$$

subject to

$$\begin{aligned} \theta &\in \arg\max_{\Theta_r} U(q(\theta_r), s(\theta_r), y(\theta_r), \theta), \quad \forall \theta \in \Theta, \text{ (IC)} \\ U(\theta) &\equiv U(q(\theta), s(\theta), y(\theta), \theta) \geq 0, \quad \forall \theta \in \Theta, \text{ (PC)} \end{aligned}$$

The reduced problem:

$$\max_{q(\theta),s(\theta),U(\theta)} E[V(q) - \theta q - c(s-\theta) - U],$$

subject to

$$\begin{aligned} U'(\theta) &= -q(\theta) + c'(s(\theta)-\theta), \quad \forall \theta \in \Theta, \text{ (IC)} \\ U(\theta) &\geq 0, \quad \forall \theta \in \Theta, \text{ (PC)} \end{aligned}$$

$$H = [V(q) - \theta q - c(s-\theta) - U]f(\theta) - \mu(q(\theta) + c'(s(\theta)-\theta)) + \tau U$$

The Model
Problem P'

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$$\begin{aligned} (V'(q) - \theta)f(\theta) - \mu(\theta) &= 0 \quad (1) \\ -c'(s(\theta) - \theta))f(\theta) + \mu c''(s(\theta) - \theta)) &= 0 \quad (2) \end{aligned}$$

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- Other conditions:

$$
\begin{aligned}
\mu'(\theta) &= f(\theta)-\tau(\theta) && (3)\\
U'(\theta) &= -q(\theta)+c'(s(\theta)-\theta) && (4)\\
\tau(\theta)U(\theta) &= 0, \tau(\theta)\geq 0, U(\theta)\geq 0 && (5)\\
\mu(\theta_0)U(\theta_0) &= 0, \mu(\theta_1)U(\theta_1)=0, \mu(\theta_0)\leq 0, \mu(\theta_1)\geq 0 && (6)
\end{aligned}
$$

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The model

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- Notation:
  - $q^*(\mu, \theta)$ and $s^*(\mu, \theta)$ denote the optimal levels, derived from (1) and (2),
  - $\mu_0(\theta)$ solution in μ to $-q^*(\theta) + c'(s^*(\theta) - \theta) = 0$.

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- Two possible cases:
  - (i) $\mu_0(\theta)$ lies above $F(\theta)$ $(\mu_0(\theta_1) \geq 1)$
  - (ii) $\mu_0(\theta)$ crosses $F(\theta)$ once from above $(\mu_0(\theta_1) < 1)$

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- $\mu(\theta) = F(\theta)$.
- From (2), all types (except θ_0) exaggerate their cost.
- From (1), the optimal level of production, $q(\theta)$, is lower than the FB level (except at θ_0).

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- This implies that if we set $\mu(\theta) = F(\theta)$, rents would be increasing for θ close to θ_1 and since $U(\theta_1) = 0$ the (PC) would be violated.
- Thus, it can be checked that:

$$\mu(\theta) = \begin{cases} F(\theta) & \text{for } \theta_0 \leq \theta \leq \theta^* \\ \mu_0(\theta) & \text{for } \theta^* \leq \theta \leq \theta_1 \end{cases}$$

where θ^* is the value of θ such that $\mu_0(\theta) = F(\theta)$. For $\theta \in [\theta^*, \theta_1]$, (PC) binds (i.e. zero rent).

Theorem

At the optimal contract, all types (except θ_0) exaggerate their cost ($s(\theta) > \theta$) and produce less than the FB amount.

Furthermore, in case (i), the (PC) is only binding at θ_1 and the optimal $q(\theta)$ and $s(\theta)$ are determined by equations (1) and (2) with $\mu(\theta) = F(\theta)$.

In case (ii), (PC) is binding on $[\theta^*, \theta_1]$. On $[\theta_0, \theta^*]$, $q(\theta)$ and $s(\theta)$ are determined as in case (i). On $[\theta^*, \theta_1]$, they are determined by equations (1) and (2) with $\mu(\theta) = \mu_0(\theta)$.

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- On $[\theta^*, \theta_1]$, the optimal output and falsification are independant of $F(\theta)$, an uncommon feature in the optimal contract literature.
- In Maggi and Rodriguez-Clare (1994), s is required to lie between θ_0 and θ_1. "Carrier constraint" case $\Rightarrow$ at the optimal contract, all types except θ_0 and θ_1 overrepresent their cost with the right interval of types displaying the highest cost θ_1.

Application

- To study the impact of the information structure on the optimal contract and welfare

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Using the formulae obtained in the general case:

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Using the formulae obtained in the general case:

- $\mu_0(\theta) = \frac{b-\theta}{1+2\alpha d}$.
- $\exists\alpha^*$ such that $\mu_0(\theta)$ crosses $F(\theta)$ iff $\alpha > \alpha^*$.

$$\alpha^* = \frac{b-2}{2d}.$$

So we can analyze separately the cases $\alpha < \alpha^*$ and $\alpha > \alpha^*$.

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- Impact of α on the agent's utility: $U'(\theta) = -q(\theta) + 2\alpha(s(\theta) - \theta)$ becomes less negative as α increases.
- Total welfare, $W(\theta) = V(q(\theta)) - \theta q(\theta) - c(s(\theta) - \theta)$ is decreasing in α.

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- When α goes to ∞, θ^* approaches zero and welfare goes to its FB level.

Conclusion

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- The standard principal-agent model with private information was extended to empasse all intermediate cases between private and public information.
- Falsification can be beneficial because it helps to reduce information rents in spite of the waste of resources involved.