Robust Inference in Communication Games with Partial Provability

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Basic concept

- Partial provability: IP only be able to prove some (but not all) of what he knows.
- Conflicting preference: IPs disagree about the relative desirability of possible actions by DM
- Robust inference rule: leads to correct inference given any equilibrium responses of IP and any preferences in $\mathscr{P}$.

Our focus:

- How much provability is required to achieve full and robust revelation in sequential message-sending games?

Literature Review

- **With no provability**: Spence "signaling game" Crawford and Sobel (1982), Mechanism design
- **With complete provability**: Milgrom(1981), Milgrom and Roberts(1986). Their insight: "Skepticism in the face of vagueness"
- **With partial provability**: small, single IP case.

Key findings:

- Condition on the structure of provability:
  - Sufficient condition: Refutability
  - SN condition: Weak refutability

The model (1/5)

- n+1 players, N={1,...n} senders are symmetrically informed, DM is non-informed
- S: the sets of states, finite. $s \in S$
- M: set of all possible messages.
- M(s): messages which are feasible in s.
- $F(m) = \{s \mid m \in M(s)\}$:set of state in which m is feasible.

The model (2/5)

- m: prove true state is in F(m), rule out state in S\F(m)
- Rich language condition: can include a “cheap talk” claim of any $s \in F(m)$
- Sequential games: open forum & balanced sequential games
- Prior belief: $\delta^0 \in \Delta$
- Posterior belief: $\delta \in \Delta$

The model (3/5)

- $H_k(s) = [M(s)]^k$: the set of sequence of exactly k messages feasible in s
- $H^K(s) = \bigcup_{k=0}^{K} H_k(s)$: possible histories of up to K feasible messages in s
- $H^K = \bigcup_{s \in S} H^K(s)$: all possible histories.

The model (4/5)

- Rules of play: $(K, \mathscr{I})$, $\mathscr{I}: H^{K-1} \to N$
- Strategy: $\sigma_i = \{\sigma_i^s\}_{s \in S}$ where $\sigma_i^s: H_i(s) \to M(s)$
- Inference rule: $\delta: H_R \to \Delta$ $H_R = \bigcup_{s \in S} [M(s)]^K$
- PBE: for $(K, \mathscr{I})$, (σ, δ) is a PBE if:

$$\delta[h_R(h, \sigma_i^s, \sigma_{\sim i}^s)] \succcurlyeq_{i,s} \delta[h_R(h, \hat{\sigma}_i^s, \sigma_{\sim i}^s)], \quad \forall \hat{\sigma}_i^s \in \Sigma_i^s,\ h \in H_i(s),\ s \in S.$$

$$\delta(h)(s) = \begin{cases} \delta^0(s) / \sum_{s' \in S^*(h,\sigma)} \delta^0(s') & \text{if} \quad s \in S^*(h, \sigma) \\ 0 & \text{otherwise.} \end{cases}$$

The model (5/5)

Here, we focus on

- Separating equilibrium: $\delta(h_{\mathbf{R}}(\sigma^s)) = s$
- Robust inference rule: $\succcurlyeq \in \mathscr{P}$

  similar to full implementation.
- Conflicting preferences: $\mathscr{P}^*$,for every $s' \neq s$, there exist i with $s \succ_{i,s} s'$

SC for RI in an open forum(1/6)

- BUR: if for every s and $h \in H_k(s)$ with $k < K$ there is $m_{s,h} \in M(s)$ such that $\delta(h \cdot m_{s,h} \cdot h') = s$ for every $h' \in H_{K-k-1}(s)$
- Intuition: believe any claim satisfying a certain burden of proof unless it is refuted by a follower.
- Key issue: what is the appropriate burden of proof ?

SC in an open forum(2/6)

- The existence of BUR depends on the structure of provability
- An IR is degenerate if $\delta(h)$ is a singleton for every h.
- Proposition 1: If δ is a degenerate BUR rule, then it is robust for $\mathscr{P}^*$ in any open forum.

SC in an open forum(3/6)

- Question: how little provability is needed for a BUR to exist?
- Complete provability:for every $\hat{S} \subseteq S$ there exist $m_{\hat{S}}$, $F(m_{\hat{S}}) = \hat{S}$
- A weaker version: $F(m_s) = \{s\}$
  - Two-way disprovability: $s \neq s' \Rightarrow M(s) \not\subseteq M(s')$
  - Full report condition: $F(m_s^*) = \bigcap_{m \in M(s)} F(m)$

SC in an open forum(4/6)

- One-way diaprovability: $s \neq s' \Rightarrow M(s) \neq M(s')$
- Refutability: if for every s' and every $s \notin S^*(s')$,we have $T(s) \not\subseteq M(s')$

  Define:

  $S^*(s) = \{s' \neq s \mid M(s') \subseteq M(s)\}$, $T(s) = M(s) \setminus \left[\bigcup_{s' \in S^*(s)} M(s') \right]$
- Intuition: guarantees that a message $m \in T(s) \setminus M(s')$ is available to claim s while refuting any outstanding s'

SC in an open forum(5/6)

An example:

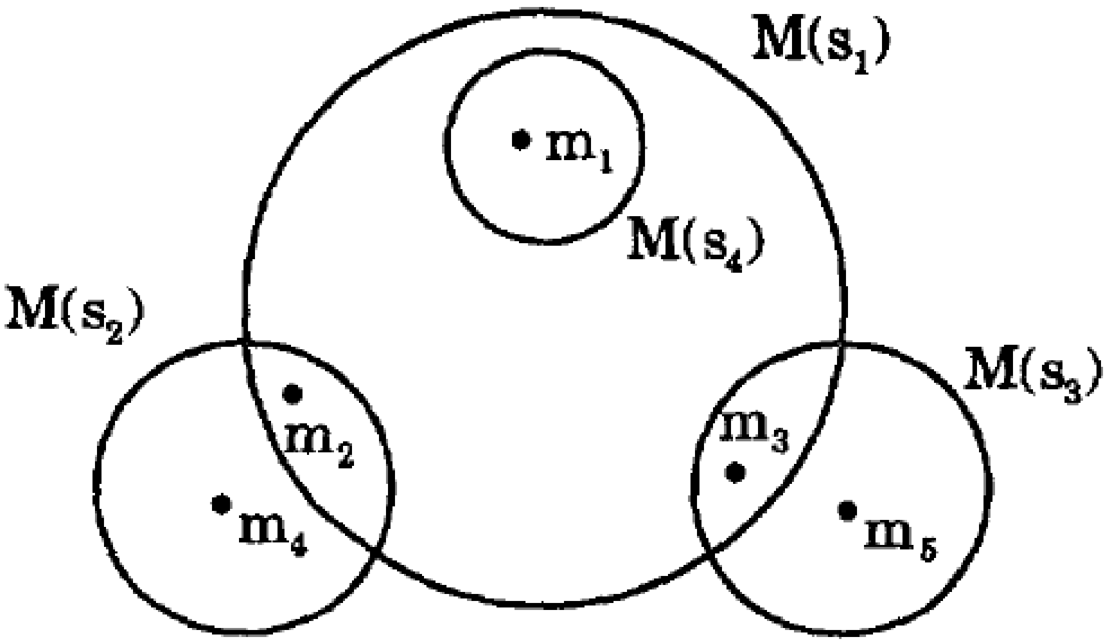

FIG. 1. Refutability without two-way disprovability or full reports.

SC in an open forum(6/6)

- Proposition 3: a degenerate BUR rules exists if the message sets satisfy refutability.
- Refutability guarantees that senders can always replace any false trustworthy claim on the table at their turn with a trustworthy claim of the true state.

NC and SC for RI in sequential games(1/6)

- Proposition 4. In any SG $(K, \mathscr{I})$, one-way disprovability is necessary for the existence of RI rule for any $\mathscr{P}$ containing at least one state independent preference profile.
- Forceable: in $(K, \mathscr{I})$, if for every s and $\sigma^{s}_{\sim i} \in \Sigma^{s}_{\sim i}$, there exist $\sigma^{s}_{i} \in \Sigma^{s}_{i}$ such that $\delta(h_{\mathbf{R}}(\sigma^{s}_{i}, \sigma^{s}_{\sim i})) = s$

NC and SC for RI in sequential games(2/6)

- Proposition 5. δ is a RI rule for $\mathscr{P}_i^*$ for a SG $(K, \mathscr{I})$ only if it is forceble. If a forceable δ exists for $(K, \mathscr{I})$, then a forceable, degenerate δ exist and is a RI rule for $\mathscr{P}^*$ for $(K, \mathscr{I})$
- A SG is balanced if for every s, every $h \in [M(s)]^K$ such that F(h) is not a singleton, and every i , there is $\mathscr{I}(h') = i$.

NC and SC for RI in sequential games(3/6)

- If a RI rule exists for $\mathscr{P}_I^*$ for $(K, \mathscr{I})$, then $(K, \mathscr{I})$ is balanced.
- Proposition 6. a forceable inference rule for an Open forum is BUR. Any BUR is forceable for any balanced SG.
- Corollary: a degenarate BUR is a RI for $\mathscr{P}^*$ for open forum, a RI for $\mathscr{P}^*$ for open forum must be a BUR

NC and SC for RI in sequential games(4/6)

- Weak refutability: if $\tau^*(s) \neq \varnothing$ for all s.
- For any s and h with $s \in F(h)$, let $\tau_1(s \mid h) = M(s)$

Recursively define:

$$\tau_k(s \mid h) = \{m \in M(s) \mid \nexists s' \in F(h \cdot m), s' \neq s, \text{ with } \tau_{k-1}(s' \mid h \cdot m) \subseteq M(s)\}$$

- Intuition : use $\tau_k(s \mid h) \backslash M(s')$ to refute s' and claim s.

  Prevent senders from making claims in ways which block later feasible challenges.

NC and SC for RI in sequential games(5/6)

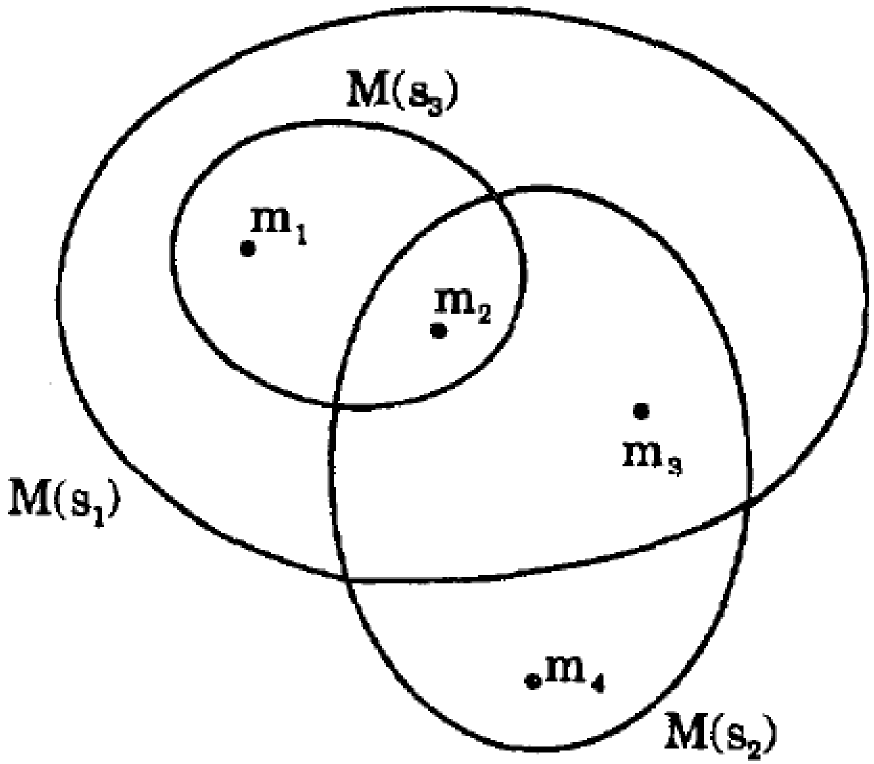

FIG. 3. Weak refutability without refutability.

NC and SC for RI in sequential games(6/6)

- Prop7. A degenerate BUR exist iff weak refutability is satisfied.
- Corollary3, if K=n=2 and $T(s) \neq \varnothing$ for all s, then a RI for $\mathscr{P}^*$ for any balanced SG exist.
- Prop 8: Refutability implies WR.
- Intuition: there may exist redundancies in refutability. WR may require a sender to rule out a state for "helping out" a later sender.

Comments and extensions:

- 1, optimal mechanisms
- 2, RI with more information about preferences
- 3, RI without conflicting preferences