Communication equilibria with partially verifiable types

Forges and Koessler

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Contribution

They characterize the set of equilibria of Bayesian games with general communication systems in which some messages can be verified.

"Chicken" game with Communication

	Stay	Move
Stay	$(0,0)$	$(7,2)$
Move	$(2,7)$	$(6,6)$

This game has three equilibria: $(Stay, Move)$, $(Move, Stay)$ and $(1/3 * Stay + 2/3 * Move, 1/3 * Stay + 2/3 * Move)$

Improving game outcome

- Using a publicly observable signal: Sunspot
- Using a communication system: Mediator recommends $\begin{pmatrix} 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

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Bayesian Game

The Bayesian (finite) game in its strategic-form is

$$G = \langle N, (A_i)_{i \in N}, (T_i)_{i \in N}, p, (u_i)_{i \in N} \rangle,$$

where

- $N = \{1, \ldots, n\}$: Set of players;
- A_i : Player's i set of possible actions;
- T_i : Player's i set of possible types;
- $p \in \Delta(T)$: Common prior over types
- $u_i : A \times T \to \mathbb{R}$: Player's i payoff function

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Communication System

The communication system is denoted

$$c = \langle (R_i)_{i \in N}, (S_i)_{i \in N}, (M_i)_{i \in N}, K, (\nu^k)_{k=0,1,\ldots,K} \rangle$$

where

- $R_i : T_i \to \mathcal{R}_i$: Reporting correspondences
- S_i : Cheap-talk signals
- M_i : Outputs
- K : Number of communication period
- $\nu^k : M^k \times \mathcal{R}^k \times S^k \to \Delta(M)$: Transition probability

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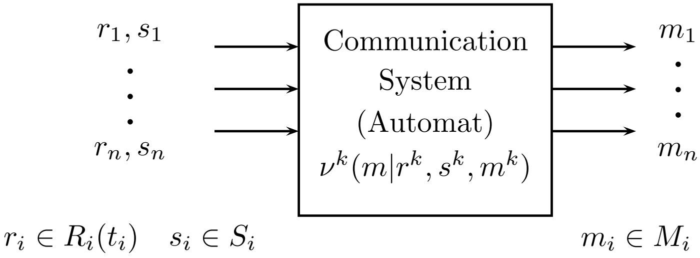

Extended Bayesian Game

A new game G_c is obtained by appending c to G.

Timing

- At period $k = 0$, outputs m^0 are privately sent before players learn their type;
- Players privately are informed about their types;
- At the beginning of period $k = 1, \ldots, K$, each player i sends confidential inputs (r_i^k, s_i^k);
- At the end of period k, each player i privately receives output m_i^k and
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Strategies in G_c

A behavioral strategy for player i in G_c is $((\sigma_i^k)_{k=1,\ldots,K}, \delta_i)$ where:

$$\sigma_i{}^k : M_i^k \times \mathcal{R}_i{}^{k-1} \times S_i^{k-1} \times T_i \to \Delta(\mathcal{R}_i \times S_i)$$

satisfying $\sigma_i{}^k(r_i^k, s_i^k|\cdot, t_i) = 0$ when $r_i^k \notin R_i(t_i)$.

$$\delta_i : M_i^{K+1} \times \mathcal{R}_i{}^K \times S_i{}^K \times T_i \to \Delta(A_i)$$

Notation: $(\sigma, \delta) = (\sigma_i, \delta_i)_{i \in N}$ where $\sigma_i = (\sigma_i{}^k)_{k=1,\ldots,K}$

Equilibrium Outcome

Each profile (σ, δ) generates:

- an *outcome* $\mu : T \to \Delta(A)$ and
- an expected payoff $\sum_{t \in T} p(t) \sum_{a \in A} \mu(a|t) u_i(a, t)$

Definition

The outcome generated by a (Bayesian) Nash equilibrium of G_c is called an *equilibrium outcome* of G_c.

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Certification Equilibrium of G

Definition

A *certification equilibrium* of G is a Nash equilibrium of the extended game G_c obtained by adding a communication system c to G. The set of certification equilibrium outcomes is denoted by $\mathcal{E}$.

Characterization of $\mathcal{E}$

It can be shown that $\mathcal{E}$ coincides with set outcomes of extended games obtained by adding a communication system with:

- one period: $K = 1$
- no initial output: ν^0 is degenerated
- no cheap-talk input: S is singleton
- $M = A$ and $R_i(t_i) = \{t_i\}$
- each player follows the recommendation of the mediator

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Characterization of $\mathcal{E}$

A version of the Revelation Principle:
Any **certification equilibrium** *is outcome equivalent to a* **truthful certification equilibrium**.

However

- the generated set of truthful certification equilibria is too much large to be interesting, and is not appropriate for most applications.

Reasons

- Players may have the right to remain silent or
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Example

	a^1	a^2
t^1	$(8, 2)$	$(10, 0)$
t^2	$(10, 0)$	$(8, 2)$

Consumer: $T_1 = \{t^1, t^2\}$, A_1 singleton and $p(t^1) = p(t^2) = 1/2$
t^i = consumer has 10 units of commodity i

Government: $A_2 = \{a^1, a^2\}$ and T_2 singleton
a^i = gvt deducts taxes of 20% on commodity i

Example

Let $R_1 = \{\{t^1\}, \{t^2\}\}$

The set of equilibrium outcomes using R_1 is characterized by $\mu(a^1|t^1) = \mu(a^2|t^2)$
$\mathcal{E}$ is characterized by $\mu(a^1|t^1) \geq \mu(a^2|t^2)$

R-certification equilibria

Focus on smaller set of equilibria: Equilibria of extended games which communication systems have fixed reporting correspondence R.

Definition

A *R-certification equilibrium* of G is a Nash equilibrium of the extended game G_c obtained by adding a R-communication system c to G. The set of R-certificate outcomes is denoted $\mathcal{E}(R)$

Canonical Representations

Objective

Characterization of $\mathcal{E}(R)$ in a tractable way.

Definition

Certifiability configuration

$$Y_i : T_i \to \mathcal{Y}_i \quad i = 1, \ldots, n$$

where the set of certificates is $\mathcal{Y}_i \subseteq 2^{T_i} \setminus \{\emptyset\}$ and

$$Y_i(t_i) = \{y_i \in \mathcal{Y}_i | t_i \in y_i\} \neq \emptyset$$

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Canonical Representations

Definitions

The *closure of* Y is a certifiability configuration $\bar{Y}$ where $\bar{Y}_i(t_i) = \{y_i \in \bar{\mathcal{Y}}_i | t_i \in y_i\}$ and $\bar{\mathcal{Y}}_i$ is the smallest set containing $\mathcal{Y}_i$ which is closed under intersection.

The smallest certifiable event for player i with type t_i is

$$\text{Mini } Y_i(t_i) \equiv \bigcap_{y_i \in Y(t_i)} y_i$$

Canonical Representations

Example

Consider $T_i = \{A, B, C\}$ and $\mathcal{Y}_i = \{A, C, \{A, B\}, \{B, C\}\}$.

- $Y_i(A) = \{A, \{A, B\}\}$, $Y_i(B) = \{\{A, B\}, \{B, C\}\}$ and $Y_i(C) = \{C, \{B, C\}\}$
- Mini $Y_i(A) = \{A\}$, Mini $Y_i(B) = \{B\}$ and Mini $Y_i(C) = \{C\}$
- $\bar{\mathcal{Y}}_i = \{A, B, C, \{A, B\}, \{B, C\}\}$
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Canonical Representations

Let $R = (R_i)_i$ be an arbitrary profile of reporting correspondences. This profile generates an unique certifiability configuration Y^R where

$$Y_i^R(t_i) \equiv \{R_i^{-1}(r_i) | r_i \in R_i(t_i)\}$$
$$R_i^{-1}(r_i) \equiv \{t_i \in T_i | r_i \in R_i(t_i)\}$$

hence

$$\mathcal{Y}_i^R \equiv \bigcup_{t_i \in T_i} Y_i^R(t_i)$$

Canonical Representations

Example

$T_1 = \{t^1, t^2, t^3\}$, $A_2 = \{a^1, a^2\}$ and T_2, A_1 singletons

Reporting correspondence R
$R(t_1) = \{r, r'\}$, $R(t_2) = R(t_3) = \{r, r', r''\}$

R generates cerfifiability configuration $Y_1^R(t)$ given by

$$Y_1^R(t_1) = \{T_1\} \quad Y_1^R(t_2) = Y_1^R(t_3) = \{\{t^2, t^3\}, T_1\}$$
$$\mathcal{Y}_1^R = \{\{t^2, t^3\}, T_1\}$$

Canonical Representations

Definition
Given a certifiability configuration Y and its closure $\bar{Y}$, we define a canonical $\bar{Y}$-communication system as a $\bar{Y}$-communication system such that $S = T$, $M = A$, $K = 1$ and ν^0 degenerated.

Canonical Representations

Definition

A canonical $\bar{Y}$-certification equilibrium of G is a Nash equilibrium of the extended game G_c obtained by adding a canonical $\bar{Y}$-communication system c to G, and

- each player i with type t_i certifies Mini $Y_i(t_i)$,
- truthfully reveals his type and
- follows mediator's recommendation.

The set of canonical $\bar{Y}$-certification equilibrium outcomes is denoted $\mathcal{E}^*(\bar{Y})$

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The set $\mathcal{E}^*(Y)$ is simply characterized by all recommendation $\nu^* : \bar{\mathcal{Y}}_i \times T \to \Delta(A)$ satifying:

$$\sum_{t_{-i} \in T_{-i}} p(t_{-i}|t_i) \sum_{a \in A} \nu^*(a|\text{Mini } Y(t), t) u_i(a; t)$$

$$\geq \sum_{t_{-i} \in T_{-i}} p(t_{-i}|t_i) \sum_{a \in A} \nu^*(a|(\text{Mini } Y_{-i}(t_{-i}), y_i), (t_{-i}, t_i^{'})) u_i(a_{-i}, d_i(a_i); t)$$

for all $i \in N$, $t_i, t_i^{'} in T_i$, $y_i \in \bar{Y}_i(t_i)$ and $d_i : A_i \to A_i$.

Main Result

Theorem
$\mathcal{E}(R) = \mathcal{E}^*(\bar{Y}^R)$ for all profiles of reporting correspondences R.

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t^2	$(0,0)$	$(1,1)$
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t^3	$(0,0)$	$(1,1)$

$R(t_1) = \{r, r'\}$, $R(t_2) = R(t_3) = \{r, r', r''\}$

$$Y_1^R(t_1) = \{T_1\} \quad Y_1^R(t_2) = Y_1^R(t_3) = \{\{t^2, t^3\}, T_1\}$$

$$\mathcal{Y}_1^R = \{\{t^2, t^3\}, T_1\}$$

Canonical Representations

Example

	a^1	a^2
t^1	$(0,1)$	$(1,0)$
t^2	$(0,0)$	$(1,1)$
t^3	$(0,0)$	$(1,1)$

$$Y_1^R(t_1) = \{T_1\} \quad Y_1^R(t_2) = Y_1^R(t_3) = \{\{t^2, t^3\}, T_1\}$$
$$\mathcal{Y}_1^R = \{\{t^2, t^3\}, T_1\}$$

The complete information outcome is implemented by $\nu^* : \mathcal{Y}^R \times T_1 \to \Delta(A)$ satisfying:

$$\nu^*(a^2|(\{t^2, t^3\}, t^2)) = \nu^*(a^2|(\{t^2, t^3\}, t^3)) = 1$$
$$\nu^*(a^1|(y, t)) = 1 \text{ for all other input } (y, t) \in \mathcal{Y}^R \times T_1$$

Canonical Representations

One-period communication systems

Let $\mathcal{E}(R|K=1)$ be the set of all one-period R-certification equilibrium outcomes.

Which conditions on R guarantee that

$$\mathcal{E}(R|K=1) = \mathcal{E}(R) \quad ?$$

Canonical Representations

Example

	a^1	a^2	a^3
t_d^1	(2, 1)	(0, 2)	(1, −2)
t_h^1	(2, 1)	(0, 2)	(1, −2)
t_d^2	(2, 1)	(1, −2)	(0, 2)
t_h^2	(2, 1)	(1, −2)	(0, 2)

Reporting correspondences

$$R(t_h^1) = \{t^1\} \quad R(t_h^2) = \{t^2\}$$
$$R(t_d^1) = R(t_d^2) = \{t^1, t^2\}$$

Certifiability configuration generated by R:

$$\mathcal{Y}^R = \{\{t_h^1, t_d^1, t_d^2\}, \{t_h^2, t_d^1, t_d^2\}\}$$

Canonical Representations

Example

The R-certification equilibrium outcome

$$\mu(a^1|t_d^1) = \mu(a^1|t_d^2) = \mu(a^2|t_h^1) = \mu(a^3|t_h^2) = 1$$

cannot be achieved using any one-period R-communication system.

Reason

Dishonest types cannot be distinguished form honest one using an one-period R-communication system.

Canonical Representations

Example

The R-certification equilibrium outcome

$$\mu(a^1|t_d^1) = \mu(a^1|t_d^2) = \mu(a^2|t_h^1) = \mu(a^3|t_h^2) = 1$$

cannot be achieved using any one-period R-communication system.

Reason

Dishonest types cannot be distinguished form honest one using an one-period R-communication system.

Canonical Representations

Definition

A certifiability configuration Y, or an associated profile of reporting correspondences R such that $Y = Y^R$, satisfies the *Minimal Closure Condition* (MCC) if Mini $Y_i(t_i) \in Y_i(t_i)$ for all $i \in N$ and $t_i \in T_i$.

Canonical Representations

Theorem

If R satisfies *MCC*, then $\mathcal{E}(R|K=1) = \mathcal{E}(R)$.

An Alternative Representation

Definition

Given any profile R of reporting correspondences, let

$$R_i^*(t_i) \equiv \{s_i \in T_i | \operatorname{Min}_i Y_i^R(s_i) \in \bar{Y}_i^R(t_i)\}$$

and $\mathcal{E}^{\#}(R^*)$ be the set of Y^{R^*}-equilibrium outcomes in which ν^0 is degenerated, S is a singleton, $M = A$ and equilibrium strategies are truthful and obedient.

Theorem

$\mathcal{E}(R) \subseteq \mathcal{E}^{\#}(R^*)$ for all R.

An Alternative Representation

Definition

Given any profile R of reporting correspondences, let

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Theorem

$\mathcal{E}(R) \subseteq \mathcal{E}^{\#}(R^*)$ for all R.