Good News and Bad news: Representation Theorem and Applications

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- Intuition: to use signals on an unobservable variable sufficiently informative to take actions based on.
- Instrument: Monotonicity of the Likelihood Ratio Property (MLRP) for conditional densities .

Plan

- Theory and Application to the Moral Hazard Model

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- Application to the Persuasion game (model studied more deeper by Malin)

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- Let X be the set of possible signals about $\widetilde{\theta}$.
- Let $f(x/\theta)$, the confitional density (or probability mass) function on X when $\widetilde{\theta}$ takes the particular value θ.
- We define the relation "more favorable than" such that:
  a signal x is more favorable than y if for every nondegenerate prioir distribution G for θ, the posterior distribution $G(./x)$ dominates the posterior distribution $G(./y)$ in the sense of strict first-order stochastic dominance.

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- A signal x is observed with the likelihood $f(x/\theta)$ when $\widetilde{\theta} = \theta$.
- The Bayes'rule in odds form for x is such that:

$$\frac{g(\overline{\theta}/x)}{g(\theta/x)} = \frac{f(x/\overline{\theta})}{f(x/\theta)} \frac{g(\overline{\theta})}{g(\theta)} \qquad (1)$$

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- We use this bayes'rule also for y.

- We assume that $\theta < \overline{\theta}$ and $g(\theta) = g(\overline{\theta}) = \frac{1}{2}$.

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- As higher values of the signal make the higher parameter value relatively more likely.
- Then, it follows that:

$$\frac{f(x/\overline{\theta})}{f(x/\theta)} > \frac{f(y/\overline{\theta})}{f(y/\theta)} \tag{2}$$

Theorem

x is more favorable than y if and only if for every $\theta < \overline{\theta}$,

$$f(x/\overline{\theta})f(y/\theta) - f(x/\theta)f(y/\overline{\theta}) > 0.$$

Theorem

The family of densities$\{f(./\theta)\}$ has the strict MLRP iff $x > y$ implies that x is more favorable than y, i.e., that the posterior distribution $G(./x)$ dominates the posterior distribution $G(./y)$ in the sense of strict first-order stochastic dominance .

- When comparing two conditional functions, the MLRP establishes that a comparatively high outcome is always more likely under one distribution amond the pair. This implies the FOSD is both necessary and sufficient on domain-conditioned distribution functions.

- Note that two signals are called **comparable** if they are equivalent (the posterior beliefs about $\widetilde{\theta}$ are identical) or if one is more favorable than the other.

Theorem

Let X be general and suppose that any two signals in X are comparable. Then, there exists a function $H : X \to \mathbb{R}$ such that $H(\widetilde{x})$ is a sufficient statistic for $\widetilde{x}$ and such that the densities of $H(\widetilde{x})$ have the strict MLRP.

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- This sufficient statisitc captures all information that is relevant to guessing the values of the unobservable parameters.

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Theorem

Let $\widetilde{x}$ be a random variable whose densities have the strict MLRP. For any two intervals $[a, b]$ and $[c, d]$ with $a \geq c$ and $b \geq d$, where at least one inequality is strict, the signal $\{\widetilde{x} \in [a, b]\}$ is more favorable than $\{\widetilde{x} \in [c, d]\}$.

- Let θ an agent's effort influencing the profit of a venture π as well as the random state of nature $\widetilde{\alpha}$ such that:$\widetilde{\pi} = \pi(\widetilde{\alpha}, \theta)$. This effort always improved profits $(\partial\pi/\partial\theta > 0)$, but there are diminishing returns to effort $(\partial^2\pi/\partial\theta^2 < 0)$.

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- Let $V(x, \theta) = U(x) - \theta$, the agent's payoffs that is an increasing function of his wealth x and a decreasing function of effort θ.

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- We assume that the agent is risk averse, i.e., $U" < 0$.
- The principal's payoff $G(x)$ is such that $G' > 0$ and $G" \leq 0$.
- $s(.)$ is the sharing rule depending only on π; $\widetilde{\alpha}$ and θ are unobservable by the principal.

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$$\frac{G'(\pi - s(\pi))}{U'(s(\pi))} = b + c\frac{f_\theta(\pi/\theta^*)}{f(\pi/\theta^*)}$$

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- Greater profits are evidence of greater efforts by the agent, then, the sharing rule should slope upwards to provide the correct incentives.

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- The salesman's payoff is his commission, which is some increasing function of q. It is assumed that F is bounded, increasing, concave, and differentiable, and that $F'(0) = +\infty$.
- Let the salesman have N pieces of data about his product, represented by $\widetilde{x} = (\widetilde{x}_1, ..., \widetilde{x}_N)$.

- The report S by the salesman is interpreted as an assertion that $\widetilde{x} \in S$.

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- The purchase decison is q.
- The purchasing strategy $b(S)$ depend on the salesman report.
- Concept of the sequential equilibrium such that the buyer interprets the reports he receives and then concludes that $\widetilde{x} \in C(s)$.

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  - For every $x \in \mathbb{R}^N$, $r(x)$ solves $\max_s b(S)$, subject to $x \in S$.
  - For every S in the range of r, $c(S) = r^{-1}(S)$.

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- Consequently, r is a strategy of full disclosure if $E[\widetilde{\theta} / \widetilde{x} = x] \equiv E[\widetilde{\theta} / r(\widetilde{x}) = r(x)]$.

Theorem

At every sequential equilibrium of the sales encounter game, the salesman uses a strategy of full disclosure.

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Theorem

The modified sales encounter game has a sequential equilibrium in which the salesman always reports the k most favorable observations.